Much Ado About N...atrium:

Modelling Tissue Sodium As A Highly Sensitive Marker Of Subclinical And Localised Oedema

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SUPPLEMENTAL MATERIAL

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Abbreviations

[Na⁺]_i = intracellular Na⁺ concentration (mmo/l)

[Na⁺]_e = extracellular Na⁺ concentration (mmol/l)

 $[Na^{+}]_{T}$ = total Na⁺ concentration in tissue (mmol/l)

 $[Na^{+}]_{OT}$ = total Na^{+} concentration in oedematous tissue (mmol/l)

 $[K^+]_i$ = intracellular K^+ concentration (mmol/l)

 $[K^+]_e$ = extracellular K^+ concentration (mmol/l)

 $[K^{+}]_{T}$ = total K^{+} concentration in a tissue (mmol/l)

[K⁺]_{OT} = total Na⁺ concentration in oedematous tissue (mmol/l)

V_i = volume of intracellular water solution in tissue

V_e = volume of extracellular water solution in tissue

 V_T = volume of total (extracellular + intracellular) water solution in tissue

ECV% = extracellular volume fraction (%) = $100 \cdot V_e/V_T$

OE% = percentage of oedema added to a tissue

Model for expected total concentration of Na⁺ and K⁺ in a tissue

For any tissue, considered as a sum of two different water solutions:

Na⁺ total moles = Na⁺ extracellular moles + Na⁺ intracellular moles

Therefore:

$$[Na^+]_T \cdot V_T = [Na^+]_e \cdot V_e + [Na^+]_i \cdot V_i$$

or

$$[Na^{+}]_{T} = [Na^{+}]_{e} \cdot V_{e}/V_{T} + [Na^{+}]_{i} \cdot V_{i}/V_{T}$$

= $[Na^{+}]_{e} \cdot ECV\%/100 + [Na^{+}]_{i} \cdot (1-ECV\%)/100$

Assuming a convenient reference volume of tissue containing 1 L of total (intracellular + extracellular) water solution, absolute Na^+ tissue content (moles) numerically coincides (\approx) with $[Na^+]_T$.

Similarly, for potassium:

$$[K^{+}]_{T} = [K^{+}]_{e} \cdot ECV\%/100 + [K^{+}]_{i} \cdot (1-ECV\%)/100$$

For a similar reference volume of tissue as above, absolute K^+ tissue content (moles) $\approx [K^+]_T$

The last two equations were used to generate the model for expected total concentration of Na⁺ and K⁺ for any tissue with 15% < ECV% < 85% in its "baseline" conditions (Figure, left panel, open symbols).

Model for expected total concentration of Na⁺ and K⁺ in an oedematous tissue

We simulated the effect of adding a fixed and biologically plausible moiety of oedema to tissues by adding 1%, 2.5% and 5% (OE%) of a solution equal in composition to the extracellular. The above percentages were defined as v/v in relation to the "baseline" volume of the water solution in the tissue, which equals 1 L in the aforementioned convenient reference tissue.

As per first equation above, absolute Na⁺ content in the oedematous tissue is:

 Na^{+} total moles $OT = (Na^{+}$ extracellular moles + Na^{+} intracellular moles)_T + Na^{+} moles in oedema

For the reference tissue, it numerically corresponds to

$$\approx [Na^{+}]_{e} \cdot ECV\%/100 + [Na^{+}]_{i} \cdot (1-ECV\%)/100 + [Na^{+}]_{e} \cdot OE\%/100$$

Na⁺ concentration in the oedematous tissue is:

[Na⁺]_{OT} = Na⁺ total moles _{OT} / total Volume

- = Na⁺ total moles _{OT} / (Volume _T + Volume _{oedema})
- = Na^+ total moles _{OT} / (1L + 1L · OE%/100)
- $\approx \{[Na^{+}]_{e} \cdot ECV\%/100 + [Na^{+}]_{i} \cdot (1-ECV\%)/100 + [Na^{+}]_{e} \cdot OE\%/100\}/(1L + 1L \cdot OE\%/100)\}$

Similarly, for potassium:

[Na⁺]_{OT} and [K⁺]_{OT} equations were used to generate the model for expected total concentration of Na⁺ and K⁺ in an oedematous tissue, after addition of 5% oedema (Figure, left panel, closed symbols).

Changes in Na⁺, K⁺ and water

Percentage changes (Δ %) for absolute Na⁺ content and concentration were defined, respectively, as:

 Δ % absolute Na⁺ content = (Na⁺ total moles _{OT} - Na⁺ total moles _T) · 100 / Na⁺ total moles _T

 Δ % Na⁺ concentration = ([Na⁺]_{OT} – [Na⁺]_T) · 100 / [Na⁺]_T

Percentage changes (Δ %) for absolute K⁺ content and concentration were similarly calculated but not plotted because not informative: they showed a stable decrease, numerically close to -OE%, non-significantly affected by ECV% (*data not shown*).

Percentage changes (Δ %) for water content was assumed as equal to OE%, which in fact corresponds to the v/v Δ % of the solution, rather than solvent. Of note, this approximation would at most over-estimate the Δ % for water compared to Δ % for Na⁺ and K⁺.

Biological assumptions

This mathematical model is obviously affected in absolute, but not relative, terms by changes in the baseline assumptions, i.e. $[Na^+]_i$, $[Na^+]_e$, $[K^+]_i$ and $[K^+]_e$: all these concentrations are subject to multiple and tight regulations (particularly for the intracellular site), which can act differently in different individuals and/or conditions. Nevertheless, despite minimal shifts along the vertical axes, curves behave consistently across multiple intra-extra cellular simulated conditions (*data not shown*), thus strengthening the robustness of the model.

For the purpose of this paper, the figure was generated assuming:

 $[Na^+]_e = 144 \text{ mmol/l}$ and $[K^+]_e = 4.64 \text{ mmol/l}$ (as reported in an experimental setting, for comparability (1); normal values for humans: 135-145 and 3.5-5.5, respectively(2)). $[Na^+]_l$ and $[K^+]_l$ were assumed as 10 and 140 mmol, respectively, as classically reported (2).

Supplemental references

- Titze J, Bauer K, Schafflhuber M, Dietsch P, Lang R, Schwind KH, Luft FC, Eckardt KU, Hilgers KF. Internal sodium balance in DOCA-salt rats: a body composition study. American journal of physiology Renal physiology. 2005;289:F793-802.
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